Entanglement Teleportation Through Cat-like States

Sibasish Ghosh[1]*, Guruprasad Kar[1], Anirban Roy[1]†, Debasis Sarkar[2] and Ujjwal Sen[3]‡

February 9, 2008

[1]Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B. T. Road, Calcutta 700035, India

[2]Department of Mathematics, Burdwan University, Burdwan, West Bengal, India

[3]Department of Physics, Bose Institute, 93/1 A. P. C. Road, Calcutta 700009, India

Abstract

We first consider teleportation of entangled states shared between Claire and Alice to Bob1 and Bob2 when Alice and the two Bobs share a single copy of a GHZ-class state and where *all* the four parties are at distant locations. We then generalize this situation to the case of teleportation of entangled states shared between Claire1, Claire2,....., Claire(N-1) and Alice to Bob1, Bob2,....., BobN when Alice and the N Bobs share a single copy of a Cat-like state and where again *all* the 2N parties are at distant locations.

Quantum teleportation, proposed by Bennett *et al.*(BBCJPW)[1], is a protocol by which an arbitrary qubit can be transferred (teleported) exactly from one location (where say, Alice is operating) to a possibly distant location (operated say, by Bob) by using only local operations and classical communication, without sending the particle itself. This almost impossibility is made possible by allowing Alice and Bob to *a priori* share a maximally entangled state of two qubits say,

$$\left|\phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

It is important to note that the entanglement in the channel vanishes completely after it has been used to send the qubit by using the BBCJPW protocol. Now if the sent qubit is *a priori* entangled with another qubit (possessed by Claire), it will remain so after teleportation. That is, the previously shared entanglement between Alice and Claire would now be shared between Bob and Claire.

Consider now a different situation. A source delivers an arbitrary two-qubit entangled state to Alice which must finally be shared between Bob1 and Bob2. Instead of state teleportation, Alice therefore has the task of entanglement teleportation. It would be sufficient if Alice shares a maximally entangled state with Bob1 and another with Bob2. Alice would then just teleport the two qubits using the BBCJPW protocol.[2, 3]

*res9603@isical.ac.in
†res9708@isical.ac.in
‡dhom@boseinst.ernet.in

But what if Alice shares with the Bob1-Bob2 system, less than two ebits of entanglement? Suppose for example that instead of the two maximally entangled states, Alice, Bob1 and Bob2 share the GHZ state[4]

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

Would the same feat be possible now? Gorbachev and Trubilko[5] have considered this case and shown that if Alice knows that the state has been prepared in the Schmidt basis $\{|0'0''\rangle, |1'1''\rangle\}$, i.e., if Alice knows that the state is of the form

$$|\chi\rangle = \alpha\,|0'0''\rangle + \beta\,|1'1''\rangle$$

with known $|0'0''\rangle$ and $|1'1''\rangle$ but unknown Schmidt coefficients α, β, then this state can be made to share between Bob1 and Bob2. In this paper we simplify their protocol. Shi *et al.*[6] generalized this situation to the case in which the state $|\chi\rangle$ is shared between *two separated parties* Alice and Claire.[7] The protocol of Shi *et al.* is probabilistic as in their case Alice and the two Bobs share the state

$$|GHZ'\rangle = a\,|000\rangle + b\,|111\rangle$$

instead of a GHZ. Here we show that even if Alice and the two Bobs share the state

$$|ghz\rangle = \frac{1}{\sqrt{2}}(|0\phi 0\rangle + |1\phi' 1\rangle)$$

where $|\phi\rangle$ *and* $|\phi'\rangle$ *are not necessarily orthogonal*, it is possible for Alice and Claire to make the two Bobs share the state

$$|\chi'\rangle = \alpha\,|\phi 0'\rangle + \beta\,|\phi' 1'\rangle$$

Our protocol is independent of the ones in ref.[8] and much simpler. More important is the fact that our protocol is generalizable to the N-party situation. We also touch the probabilistic case in both situations.

Suppose a source prepares the state $|\chi\rangle$ (with known $|0'0''\rangle$ and $|1'1''\rangle$ but unknown α, β) and delivers it to Alice who wants to make it shared between Bob1 and Bob2 through a GHZ state which she shares with the Bobs. This situation has been considered in ref.[5]. We simplify their protocol and show that $|\chi\rangle$ can be made to share between the two Bobs by simply using the BBCJPW protocol. Indeed $|\chi\rangle$ is essentially a qubit. What is important is that there is no nonlocal operation involved between the two Bobs in the protocol. Let us elaborate.

First Alice transforms $|\chi\rangle$ to

$$|\xi\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$$

which is possible as the Schmidt basis $\{|0'0''\rangle, |1'1''\rangle\}$ is known. The combined state $|\xi\rangle_{12}\,|GHZ\rangle_{AB_1B_2}$ may be written as

$$\frac{1}{2}[\left|\phi^+_{GHZ}\right\rangle_{12A} \otimes (\alpha\,|00\rangle + \beta\,|11\rangle)_{B_1B_2} + \left|\phi^-_{GHZ}\right\rangle_{12A} \otimes (\alpha\,|00\rangle - \beta\,|11\rangle)_{B_1B_2}$$

$$+ \left|\psi^+_{GHZ}\right\rangle_{12A} \otimes (\alpha\,|11\rangle + \beta\,|00\rangle)_{B_1B_2} + \left|\psi^-_{GHZ}\right\rangle_{12A} \otimes (\alpha\,|11\rangle - \beta\,|00\rangle)_{B_1B_2}]$$

where

$$\left|\phi^\pm_{GHZ}\right\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)$$

$$\left|\psi^\pm_{GHZ}\right\rangle = \frac{1}{\sqrt{2}}(|001\rangle \pm |110\rangle)$$

Alice now performs a projection-valued measurement on her three qubits (1, 2 and A) with the projection operators

$$P_1^{GHZ} = P\left[\left|\phi^+_{GHZ}\right\rangle\right],\ P_2^{GHZ} = P\left[\left|\phi^-_{GHZ}\right\rangle\right]$$

$$P_3^{GHZ} = P\left[\left|\psi^+_{GHZ}\right\rangle\right],\; P_4^{GHZ} = P\left[\left|\psi^-_{GHZ}\right\rangle\right]$$

$$P_5^{GHZ} = I - \sum_{i=1}^{4} P_i^{GHZ}$$

and communicates the result to Bob1 and Bob2. If P_1^{GHZ} clicks, the Bobs are to do nothing. They would then already share the state $|\xi\rangle$. If P_2^{GHZ} clicks, Bob2 does nothing but Bob1 applies σ_z on his particle. If P_3^{GHZ} clicks, both of them apply σ_x i.e., they apply $\sigma_x \otimes \sigma_x$. And if P_4^{GHZ} clicks they apply $\sigma_x \otimes i\sigma_y$. P_5^{GHZ} would never click. Finally, Bob1 and Bob2 share the state $|\xi\rangle$ on which they apply $U_1 \otimes U_2$ to transform it to $|\chi\rangle$ where U_1 is the unitary operator that transforms $|0\rangle \to |0'\rangle$ and $|1\rangle \to |1'\rangle$ and U_2 the unitary operator that transforms $|0\rangle \to |0''\rangle$ and $|1\rangle \to |1''\rangle$. Note that throughout the process there is no nonlocal operation involved between Bob1 and Bob2.

Shi *et al.*[6] have generalized this situation to the case in which two separated parties Alice and Claire share the state $|\chi\rangle$. We show that even in the case in which Alice and the two Bobs share the state

$$|ghz\rangle = \frac{1}{\sqrt{2}}(|0\phi 0\rangle + |1\phi' 1\rangle)$$

where $|\phi\rangle$ and $|\phi'\rangle$ are not necessarily orthogonal, it is possible for Alice and Claire to make the two Bobs share the state

$$|\chi'\rangle = \alpha\,|\phi 0'\rangle + \beta\,|\phi' 1'\rangle$$

Note that $|ghz\rangle$ is a state of the "GHZ-class"[8].

The initial combined state is

$$|\chi'\rangle_{12}\,|ghz\rangle_{AB_1B_2} = (\alpha\,|\phi 0'\rangle + \beta\,|\phi' 1'\rangle)_{12}\,\frac{1}{\sqrt{2}}\,(|0\phi 0\rangle + |1\phi' 1\rangle)_{AB_1B_2}$$

where the particles 1 and 2 belong to Claire and Alice respectively. First of all Alice performs the unitary operation U', on the qubit 2, that transforms $|0'\rangle \to |0\rangle$ and $|1'\rangle \to e^{-i\varepsilon}\,|1\rangle$ where $\langle\phi\,|\phi'\rangle = re^{i\varepsilon}$ so that $|\chi'\rangle_{12}$ transforms to

$$|\xi'\rangle_{12} = \alpha\,|\phi 0\rangle + \beta\,|\phi'' 1\rangle$$

where $|\phi''\rangle = e^{-i\varepsilon}\,|\phi'\rangle$. Alice also applies the unitary operator, on qubit A, that transforms $|0\rangle \to |0\rangle$ and $|1\rangle \to e^{-i\varepsilon}\,|1\rangle$ so that $|ghz\rangle$ transforms to

$$|ghz_1\rangle = \frac{1}{\sqrt{2}}(|0\phi 0\rangle + |1\phi'' 1\rangle)$$

The state of the five particles is now

$$|\xi'\rangle_{12}\,|ghz_1\rangle_{AB_1B_2} = (\alpha\,|\phi 0\rangle + \beta\,|\phi' 1\rangle)\frac{1}{\sqrt{2}}(|0\phi 0\rangle + |1\phi'' 1\rangle)$$

which may be rewritten as

$$\frac{1}{2}[(\alpha\,|\phi\phi 0\rangle + \beta\,|\phi''\phi'' 1\rangle)_{1B_1B_2} \otimes \left|\phi^+\right\rangle_{2A} + (\alpha\,|\phi\phi 0\rangle - \beta\,|\phi''\phi'' 1\rangle)_{1B_1B_2} \otimes \left|\phi^-\right\rangle_{2A}$$

$$+\,(\alpha\,|\phi\phi'' 1\rangle + \beta\,|\phi''\phi 0\rangle)_{1B_1B_2} \otimes \left|\psi^+\right\rangle_{2A} + (\alpha\,|\phi\phi'' 1\rangle - \beta\,|\phi''\phi 0\rangle)_{1B_1B_2} \otimes \left|\psi^-\right\rangle_{2A}]$$

where

$$\left|\phi^\pm\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$\left|\psi^\pm\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Alice now conducts a projection measurement (the Bell measurement) on her two particles with the projection operators

$$P_1 = P[|\phi^+\rangle],\ P_2 = P[|\phi^-\rangle]$$
$$P_3 = P[|\psi^+\rangle],\ P_4 = P[|\psi^-\rangle]$$

After that she sends two bits of classical message to each of Bob1 and Bob2 to tell them the result of the Bell measurement.

As $\langle\phi\,|\phi''\rangle$ $(= r)$ belongs to $[0,\,1]$, there exists a unique orthonormal basis $\{|a\rangle\,,\,|\overline{a}\rangle\}$ such that

$$|\phi\rangle = \cos\frac{\theta}{2}\,|a\rangle + \sin\frac{\theta}{2}\,|\overline{a}\rangle$$

$$|\phi'\rangle = \cos\frac{\theta}{2}\,|a\rangle - \sin\frac{\theta}{2}\,|\overline{a}\rangle$$

where $\theta\,\epsilon\,[0,\,\pi/2]$. Note that θ, $|a\rangle$, $|\overline{a}\rangle$ are all known. Claire performs a projective measurement on just the basis $\{|a\rangle\,,\,|\overline{a}\rangle\}$ and communicates the result to the Bobs.

It is now straightforward to see that there always exists a *product*-unitary operation between the two Bobs, depending upon the results communicated by Alice and Claire, so that they (the Bobs) end up sharing the state $|\xi'\rangle$. If P_1 clicks in Alice's measurement, then Claire and the two Bobs share the state

$$\alpha\,|\phi\phi 0\rangle_{1B_1B_2} + \beta\,|\phi''\phi''1\rangle_{1B_1B_2}$$
$$= \alpha\cos\frac{\theta}{2}\,|a\phi 0\rangle + \alpha\sin\frac{\theta}{2}\,|\overline{a}\phi 0\rangle + \beta\cos\frac{\theta}{2}\,|a\phi''1\rangle - \beta\sin\frac{\theta}{2}\,|\overline{a}\phi''1\rangle$$
$$= \cos\frac{\theta}{2}\,|a\rangle_1\,(\alpha\,|\phi 0\rangle + \beta\,|\phi''1\rangle)_{B_1B_2} + \sin\frac{\theta}{2}\,|\overline{a}\rangle_1\,(\alpha\,|\phi 0\rangle - \beta\,|\phi''1\rangle)_{B_1B_2}$$

If after her measurement, Claire obtains the result $|a\rangle$, then the Bobs are to do nothing. On the other hand if she obtains the result $|\overline{a}\rangle$, only Bob2 is to perform a unitary operation that transforms $|0\rangle \to |0\rangle$ and $|1\rangle \to -\,|1\rangle$, which is just σ_z.

If P_2 clicks, then Claire and the Bobs share the state

$$\alpha\,|\phi\phi 0\rangle_{1B_1B_2} - \beta\,|\phi''\phi''1\rangle_{1B_1B_2}$$

$$= \cos\frac{\theta}{2}\,|a\rangle_1\,(\alpha\,|\phi 0\rangle - \beta\,|\phi''1\rangle)_{B_1B_2} + \sin\frac{\theta}{2}\,|\overline{a}\rangle_1\,(\alpha\,|\phi 0\rangle + \beta\,|\phi''1\rangle)_{B_1B_2}$$

In this case, the Bobs are to do nothing if Claire obtains $|\overline{a}\rangle$ and only Bob2 is to apply σ_z if Claire obtains $|a\rangle$.

If P_3 clicks, then the shared state is

$$\alpha\,|\phi\phi''1\rangle_{1B_1B_2} + \beta\,|\phi''\phi 0\rangle_{1B_1B_2}$$

$$= \cos\frac{\theta}{2}\,|a\rangle_1\,(\alpha\,|\phi''0\rangle + \beta\,|\phi 1\rangle)_{B_1B_2} + \sin\frac{\theta}{2}\,|\overline{a}\rangle_1\,(\alpha\,|\phi''0\rangle - \beta\,|\phi 1\rangle)_{B_1B_2}$$

Since the inner product of $|\phi\rangle$ and $|\phi''\rangle$ is real, there exists a unitary operator U'' that transforms $|\phi\rangle \to |\phi''\rangle$ and $|\phi''\rangle \to |\phi\rangle$. Irrespective of what Claire obtained, Bob1 is to apply just this operator. However Bob2 is to do nothing if Claire obtains $|a\rangle$ but to apply σ_z if Claire obtains $|\overline{a}\rangle$.

If P_4 clicks in Alice's measurement, Bob1 again applies U'' irrespective of Claire's result. And Bob2 is to do nothing if Claire obtains $|\overline{a}\rangle$ and to apply σ_z if she obtains $|a\rangle$. At the end of all these the Bobs are left with the state $|\xi'\rangle$ on which Bob2 has to perform a rotation to transform it to $|\chi'\rangle$. Precisely Bob2 has to apply $(U')^{-1}$.

Looking at the protocol described above, which is essentially a generalization of the BBCJPW protocol, it may seem that the measurement of Claire must be preceded by that of Alice, and

so Alice must communicate to Claire that her (Alice's) measurement has been performed. But that is not true. The protocol would go through irrespective of whether Alice or Claire performed the first measurement. Indeed the measurements are to be performed in two different Hilbert spaces and the corresponding projection operators would therefore commute. For example if Alice obtains $|\phi^+\rangle$ and Claire obtains $|a\rangle$, the Bobs would share the state $\alpha\,|\phi 0\rangle + \beta\,|\phi'' 1\rangle$ irrespective of who performed the first measurement.

For completeness, note that if Alice and two Bobs share the state

$$|ghz'\rangle = a\,|0\phi 0\rangle + b\,|1\phi' 1\rangle$$

the above entanglement teleportation is possible in a probabilistic manner where Alice has to change her operations in the same way as exact teleportation was changed to probabilistic teleportation in ref.[9]. Suffice it to mention that the combined state

$$|\xi'\rangle_{12}\,|ghz'_1\rangle_{AB_1B_2} = (\alpha\,|\phi 0\rangle + \beta\,|\phi'' 1\rangle)(a\,|0\phi 0\rangle + b\,|1\phi'' 1\rangle)$$

may be written as

$$\frac{1}{2}\{(\alpha\,|\phi\phi 0\rangle + \beta\,|\phi''\phi'' 1\rangle)_{1B_1B_2} \otimes (a\,|00\rangle + b\,|11\rangle)_{2A}$$
$$+(\alpha\,|\phi\phi 0\rangle - \beta\,|\phi''\phi'' 1\rangle)_{1B_1B_2} \otimes (a\,|00\rangle - b\,|11\rangle)_{2A}$$

$$+(\alpha\,|\phi\phi'' 1\rangle + \beta\,|\phi''\phi 0\rangle)_{1B_1B_2} \otimes (a\,|01\rangle + b\,|10\rangle)_{2A}$$
$$+(\alpha\,|\phi\phi'' 1\rangle - \beta\,|\phi''\phi 0\rangle)_{1B_1B_2} \otimes (a\,|01\rangle - b\,|10\rangle)_{2A}\}$$

We now go over to the N-party case. Suppose there are N Bobs, Bob1, Bob2,....., BobN and they share with Alice a Cat-like state

$$|cat\rangle_{AB_1B_2....B_N} = \frac{1}{\sqrt{2}}(|0\phi_1\phi_2....\phi_{N-1}0\rangle + |1\phi'_1\phi'_2....\phi'_{N-1}1\rangle)$$

where $|\phi_i\rangle$ and $|\phi'_i\rangle$ $(i = 1, 2, ..., N-1)$ are not necessarily orthogonal. It would then be possible to make the N Bobs share the state

$$|\chi^N\rangle = \alpha\,|\phi_1\phi_2....\phi_{N-1}0'\rangle + \beta\,|\phi'_1\phi'_2....\phi'_{N-1}1'\rangle$$

initially shared between Claire1, Claire2,....., Claire(N-1) and Alice where $|\phi_1\phi_2....\phi_{N-1}0'\rangle$ and $|\phi'_1\phi'_2....\phi'_{N-1}1'\rangle$ are known but α, β are unknown. The particles 1,2,....., N belong respectively to Claire1, Claire2,....., Claire(N-1) and Alice.

Alice first transforms $|\chi^N\rangle$ to

$$|\xi^N\rangle = \alpha\,|\phi_1\phi_2....\phi_{N-1}0\rangle + \beta\,|\phi''_1\phi''_2....\phi''_{N-1}1\rangle$$

where $|\phi''_i\rangle = e^{-i\varepsilon_i}\,|\phi'_i\rangle$, the ε_i's being given by $\langle\phi_i\,|\phi'_i\rangle = re^{i\varepsilon_i}$ $(i = 1, 2, ..., N-1)$. To effect this transformation, Alice has to apply the unitary operator, on qubit N, that transforms $|0'\rangle \rightarrow |0\rangle$ and $|1'\rangle \rightarrow e^{-i\sum_{i=1}^{N-1}\varepsilon_i}\,|1\rangle$. Alice also applies the unitary operator, on qubit A, that transforms $|0\rangle \rightarrow |0\rangle$ and $|1\rangle \rightarrow e^{-i\sum_{i=1}^{N-1}\varepsilon_i}\,|1\rangle$ so that $|cat\rangle$ transforms to

$$|cat_1\rangle = \frac{1}{\sqrt{2}}(|0\phi_1\phi_2....\phi_{N-1}0\rangle + |1\phi''_1\phi''_2....\phi''_{N-1}1\rangle)$$

The combined state of the 2N+1 particles is now

$$|\xi^N\rangle_{12....N}\,|cat_1\rangle_{AB_1B_2....B_N}$$

which may be written as

$$\left(\alpha\left|\phi_1\phi_2....\phi_{N-1}\phi_1\phi_2....\phi_{N-1}0\right\rangle+\beta\left|\phi_1''\phi_2''....\phi_{N-1}''\phi_1''\phi_2''....\phi_{N-1}''1\right\rangle\right)_{12....(N-1)B_1B_2....B_N}\otimes\left|\phi^+\right\rangle_{NA}$$

$$\left(\alpha\left|\phi_1\phi_2....\phi_{N-1}\phi_1\phi_2....\phi_{N-1}0\right\rangle-\beta\left|\phi_1''\phi_2''....\phi_{N-1}''\phi_1''\phi_2''....\phi_{N-1}''1\right\rangle\right)_{12....(N-1)B_1B_2....B_N}\otimes\left|\phi^-\right\rangle_{NA}$$

$$\left(\alpha\left|\phi_1\phi_2....\phi_{N-1}\phi_1''\phi_2''....\phi_{N-1}''0\right\rangle+\beta\left|\phi_1''\phi_2''....\phi_{N-1}''\phi_1\phi_2....\phi_{N-1}1\right\rangle\right)_{12....(N-1)B_1B_2....B_N}\otimes\left|\psi^+\right\rangle_{NA}$$

$$\left(\alpha\left|\phi_1\phi_2....\phi_{N-1}\phi_1''\phi_2''....\phi_{N-1}''0\right\rangle-\beta\left|\phi_1''\phi_2''....\phi_{N-1}''\phi_1\phi_2....\phi_{N-1}1\right\rangle\right)_{12....(N-1)B_1B_2....B_N}\otimes\left|\psi^-\right\rangle_{NA}$$

As before, Alice now performs a Bell measurement on her two qubits. And Claire(i) performs a projection measurement in the orthonormal basis $\{|a_i\rangle, |\overline{a_i}\rangle\}$ determined uniquely by

$$|\phi_i\rangle=\cos\frac{\theta_i}{2}|a_i\rangle+\sin\frac{\theta_i}{2}|\overline{a_i}\rangle$$

$$|\phi_i''\rangle=\cos\frac{\theta_i}{2}|a_i\rangle-\sin\frac{\theta_i}{2}|\overline{a_i}\rangle$$

where $\theta_i \,\epsilon\, [0,\ \pi/2]$ $(i=1,2,...,(N-1))$. Alice and the Claires now communicate their results to the Bobs. By now it is obvious that whatever be the result at Alice and the Claires, there would always exist a *product*-unitary operator between the N Bobs so that they (the Bobs) are left with the state $\left|\xi^N\right\rangle$ which may henceforth be transformed *locally* to $\left|\chi^N\right\rangle$.

Let us add that if Alice and the N Bobs share the state

$$|cat'\rangle=\alpha\left|0\phi_1\phi_2....\phi_{N-1}0\right\rangle+\beta\left|1\phi_1'\phi_2'....\phi_{N-1}'1\right\rangle$$

the above entanglement teleportation of $\left|\chi^N\right\rangle$ would be possible in a probabilistic manner.

In all the above cases of entanglement teleportation considered above, the teleported entangled state is essentially a qubit as each of them is a superposition of two different states. And in the deterministic cases, Alice shares 1 ebit of entanglement with the Bobs. Now *1 ebit may be used to teleport at most 1 qubit*. For if it were possible to teleport 2 qubits, these could *a priori* be separately entangled maximally with 2 other qubits resulting in the creation of 2 ebits of entanglement using a single ebit.[10]

To summarize, we have considered set-dependent entanglement teleportation when the available channel resource is less than what is needed for universal entanglement teleportation. In the case of teleportation of entangled states through the channel $\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{AB_1B_2}$, the pure entangled states from the plane spanned by known $|0'0''\rangle$ and $|1'1''\rangle$ can be exactly teleported by Alice to Bob1-Bob2 [5]. Note that the amount of entanglement of these states vary from 0 to 1. Shi *et al.*[6] have considered the probabilistic case when the channel is $a\,|000\rangle_{AB_1B_2}+b\,|111\rangle_{AB_1B_2}$ and when the states to be teleported are themselves shared between Alice and a distant party Claire.

We have shown that an arbitrary pure entanglement from the plane spanned by known $|\phi 0'\rangle$ and $|\phi' 1'\rangle$ (where $|\phi\rangle$ and $|\phi'\rangle$ are arbitrary but fixed, in general, non-orthogonal states) can be deterministically or probabilistically teleported (from Alice-Claire to Bob1-Bob2) by using the channel as the GHZ-class states[8] $\frac{1}{\sqrt{2}}(|0\phi 0\rangle+|1\phi' 1\rangle)$ or $a\,|0\phi 0\rangle+b\,|1\phi' 1\rangle$ respectively. Note that as the states $|\phi\rangle$ and $|\phi'\rangle$ are in general non-orthogonal states, the amount of the entanglement of the teleported states vary from 0 to e, where $e\leq 1$. And our protocol is deterministic or probabilistic depending on whether Alice shares one ebit with the Bob1-Bob2 system or less than that. It is interesting to note that although for both $|GHZ\rangle=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{AB_1B_2}$ and $|ghz\rangle=\frac{1}{\sqrt{2}}(|0\phi 0\rangle+|1\phi' 1\rangle)_{AB_1B_2}$ Alice shares 1 ebit with the Bobs, the channel between Alice and Bob2 is distillable for $|ghz\rangle$ (when $|\phi\rangle$ and $|\phi'\rangle$ are non-orthogonal) and separable for $|GHZ\rangle$ while the channels between Alice and Bob1 are separable for both.[12] This could somehow be the reason as to why 0 to 1 entanglement is *not* transferred through $|ghz\rangle$ (when $|\phi\rangle$ and $|\phi'\rangle$ are non-orthogonal) although the same is possible through $|GHZ\rangle$. We then generalized these considerations to the case in which the state to be teleported is an N-party entangled state.

U. S. thanks Dipankar Home for encouragement and acknowledges partial support by the Council of Scientific and Industrial Research, Government of India, New Delhi.

References

[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993)

[2] Note that in this way, Alice could make Bob and Claire share an *arbitrary* entangled state of two qubits, whether pure or mixed. But henceforth we shall consider entanglement teleportation of pure states only.

[3] For entanglement teleportation through two EPR channels, see J. Lee and M. S. Kim, Phys. Rev. Lett. **84**, 4236 (2000). See also H. Lu and G.-C. Guo, Phys. Lett. A **276**, 1 (2000)

[4] D. M. Greenberger, M. A. Horne and A. Zeilinger in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe* ed. M. Kafatos, (Kluwer, Dordrecht 1989); D. M. Greenberger, M. A. Horne, A. Shimony and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990); N. D. Mermin, Am. J. Phys. **58**, 731 (1990)

[5] V. N. Gorbachev and A. I. Trubilko, *Quantum teleportation of EPR pair by three particle entanglement*, quant-ph/9906110 (to be published in Russian JETF 2000)

[6] B.-S. Shi, Y.-K. Jiang and G.-C. Guo, Phys. Lett. A **268**, 161 (2000)

[7] So earlier operations performed by Alice are not possible now.

[8] W. Dur, G. Vidal and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000)

[9] T. Mor, *TelePOVM-New Faces of Teleportation*, quant-ph/9608005; T. Mor and P. Horodecki, *Teleportation via generalized measurements, and conclusive teleportation*, quant-ph/9906039; S. Bandyopadhyay, Phys. Rev. A **62**, 012308 (2000)

[10] Here we have used the fact that in teleportation of a single qubit A (possessed by say, Alice) to a possibly distant party say, Bob, through an entangled channel between Alice and Bob, an initial entanglement between A and C (possessed by say, Claire, possibly distant from both Alice and Bob) is fully transferred between C and Bob's particle. See [11]

[11] S. Ghosh, G. Kar, A. Roy and U. Sen, *Entanglement vs. Noncommutativity in Teleportation*, quant-ph/0010012

[12] Here by "channel between Alice and Bob1", we mean that part of the channel which is left when Bob2 is traced out. Similarly for "channel between Alice and Bob2".